$$(\mu_1 p_1 + \mu) \int_{1-p_1}^{1} G(y)\,dy - \mu_1 p_1^2 G(1-p_1) - \mu p_1 \geqslant 0 \ ?$$

$$= \int_{1-p_1}^{1} \left(\mu_1 p_1^2 g(y) - (\mu_1 p_1 + \mu)\bar{G}(y) \right) dy$$

< 0 pour $p_1 = \frac{1}{2}$, $\mu_1 = 1$, $\mu = 2$ et $g(u) = 2u$ (Gini)

(ex : $a_1 = 0$, $a_2 = 2$, $a_3 = 8$, $\mu_2 = 3$ avec $m = 2$ et $p_2 = \frac{1}{2}$)

pour montrer que $\frac{\partial \varphi}{\partial v_1}(\theta) > 0$

$\theta = (p_1, \dots, p_m)$

$$\frac{\partial \varphi(\theta)}{\partial v_m} = \frac{\mu_m}{\mu^2} \sum_{j=1}^{m} \int_{p_{j+1} + \dots + p_m}^{p_j + \dots + p_m} h_j(\theta, y)\,dy - \frac{1}{\mu} \left(\sum_{j=1}^{m-1} \frac{a_{j+1} - a_j}{p_j} \int_{p_{j+1} + \dots + p_m}^{p_j + \dots + p_m} g(y)\,dy + \frac{(a_{m+1} - a_m)}{p_m^2} \int_0^{p_m} y g(y)\,dy \right)$$

où $h_j(\theta, y) = \left[(a_{j+1} - a_j) \frac{p_j + \dots + p_m - y}{p_j} + a_j \right] g(y)$

$$\frac{\partial \varphi(\theta)}{\partial v_m} = \sum_{j=1}^{m-1} \int_{p_{j+1} + \dots + p_m}^{p_j + \dots + p_m} \left[\frac{a_{j+1} - a_j}{\mu p_j} \left(\frac{\mu_m}{\mu} (p_j + \dots + p_m - y) - 1 \right) + \frac{\mu_m a_j}{\mu^2} \right] g(y)\,dy$$

$$+ \int_0^{p_m} \left[\frac{a_{m+1} - a_m}{p_m \mu} \left(\frac{\mu_m}{\mu} (p_m - y) - \frac{y}{p_m} \right) + \frac{a_m \mu_m}{\mu^2} \right] g(y)\,dy$$